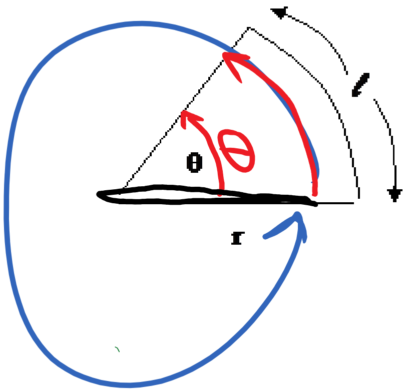
Radians and Angular Quantities

Tuesday, April 06, 2010 9:02 PM

ANGULAR MEASURES

Any linear motion or force can be used to create angular motion, an example is the creation of a torque from a force. $\tau = F \times d$ where d is the radial distance from the pivot point to the location of the force. Circular motion is the result of tangential velocity and a radially directed force and is another example of an angular measurement. When using angular measures it is VERY useful to use a system of measurements that operate in a circular fashion (using angle and radii) rather than linear measurements (X and Y axes). This is the basis of the angular measure of radians, rather than degrees. 1 rad (or radian) is defined as an angle θ which spans the length of arc ℓ where ℓ is exactly equal to the radius r or:



Θ : angular displacement ← radians

$\Theta = 2\pi$ radians full circle

angular velocity

$\frac{\Delta\Theta}{\Delta t} = \omega$

Because of this $\theta = \ell / r$ *

for a complete circle $\frac{\Delta\Theta}{\Delta t} = \frac{2\pi}{T}$

$\omega = \frac{2\pi}{T} = 2\pi f$

A tire of radius 43.18 cm (17") rotates at 9.22 Hz, find its angular velocity

$\omega = 2\pi(9.22)$

$= 57.9 \frac{rad}{sec}$

$\omega r = v$

$57.9 \times .4318 = 25 \frac{m}{s}$

$90 \frac{km}{h}$

$d = 2\pi r$?

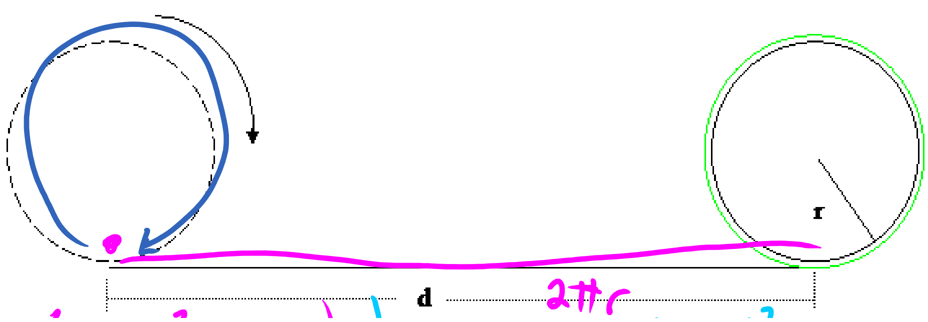
$\frac{\Delta d}{\Delta t} = v$

$\frac{2\pi r}{T} = v$

Because of this definition, a circle with circumference $2\pi r$ will have an angular span in radians of 2π rad $= 360^o$. Using this ratio a conversion can always be done from radians to degrees.

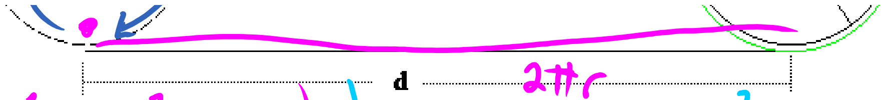
Circular Motion: distances, velocities and acceleration

As a circle turns it will trace out a linear distance on the ground based on its circumference so that $d = n\, 2\pi r$ where d is the linear distance on the ground
n is the number of turns



$\frac{\Delta d}{\Delta t} = V$

$\frac{\Delta\Theta}{\Delta t} = \omega$



$v_f^2 = v_0^2 + 2a\Delta d$ $\quad d = v_0 t + \frac{1}{2}at^2$

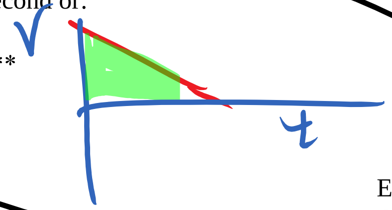
$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$ $\quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\frac{\Delta\theta}{\Delta t} = \omega$

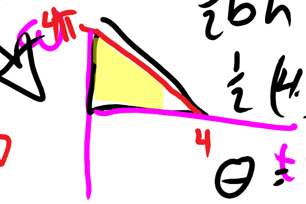
$\frac{\Delta\omega}{\Delta t} = \alpha$

As an object rotates it has what is called an angular velocity, symbolized ω, which is simply the change in angle per second or:

$\omega = \frac{\Delta\theta}{\Delta t}$ **



find angular displacement



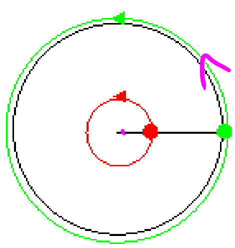
$\frac{1}{2}bh$

$\frac{1}{2}(4)4\pi$

$\theta = 8\pi$

Each point will travel the same number of degrees per second therefore the same ω.
But each has a shorter r and therefore a lesser linear velocity v = Δd / Δt or 2πr / T. Where T is one period of rotation.

Note that for any object its angular velocity is independent of r so that all points on a radial line have the same angular velocity.



$\frac{d\theta}{dt} = \omega$ $\quad \frac{d\omega}{dt} = \alpha$

$\alpha = a$

A linear velocity v is related to angular velocity ω as v = Δd / Δt. If the change in distance is a small arc of length $\Delta\ell$ then v = $\Delta\ell$ / Δt substitution for $\Delta\ell$ from eqn. * above gives us

$\tau = F$

v = rΔθ / Δt   now substitution of ** for Δθ / Δt yields:

v = rω

v = rω

Rotational Kinematics

Monday, March 30, 2015

2:08 PM

$\frac{\Delta d}{\Delta t} = \underline{V}$ $\frac{\Delta V}{\Delta t} = \underline{a}$ $V_f - V_0 = at$

$d = V_0 t + \frac{1}{2}at^2$

$V_f^2 = V_0^2 + 2a\Delta d$

Rotational Kinematics describes the motion objects in circular fashion, just like kinematics describes motion in linear (translational) fashion.

$\frac{\Delta\theta}{\Delta t} = \omega$ $\frac{\Delta\omega}{\Delta t} = \underline{\alpha}$ $\omega_f - \omega_0 = \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\tau = I\alpha$ $\frac{F_{net}}{m} = a$ $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$

Example 1: A bicycle wheel of r = 60 cm initially at rest is accelerated uniformly for 10.0 s at π/4 rad/s², what is Its angular displacement in radians, and its linear displacement in meters?

π = A-ok $\pi = 3.14$

$\omega_0 = 0$

$t = 10$

$\alpha = \frac{\pi}{4}$

$\theta = ?$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 0 + \frac{1}{2}\,\frac{\pi}{4}(10)^2$

$= \frac{100\pi}{8} = 12.5\pi$ (~~radians~~)

$d = \theta r = 12.5(\pi)(.6) = 23.6m$

Example 2: A 30 rpm record reaches its final rotational speed in 3.0 sec, determine its angular acceleration.

revolutions per minute $= \frac{30 \text{ cycles}}{60 \text{ sec}} = 0.5$ Hz

$f = \frac{1}{T}$ Hz ← $\Delta\theta = 2\pi$ $\omega = \frac{2\pi}{T}$

$2\pi f = \omega$

1 cycle ← π rad $= \omega_f$

minute 60 sec

$\frac{1}{2}$ cycle $\leftarrow \frac{\pi \text{ rad}}{\text{sec}} = \omega_f$

$\omega_0 = 0$
$t = 3$
$\alpha = ?$

$\omega_f - \omega_0 = \alpha t$ sec

$\pi - 0 = \alpha(3)$

$\frac{\pi}{3} \frac{\text{rad}}{s^2} = \alpha$

In any formula $2\pi f$ can be replaced with ω

Example 3: What effect would doubling the torque have on the linear displacement on a wheel which starts at rest?

$\frac{F_{net}}{m} = a$

$\frac{\tau}{I} = \frac{2d}{t^2 r}$

$\frac{d}{r} = \theta$

Rotational Mass

In linear motion mass is known as inertia and momentum is a direct function of m.
In rotational motion an expression of mass is known as the moment of inertia (I). For a particle (or single body) rotating at the edge of a circle $I = mr^2$.
Important shapes whose formulae for moment of inertia are:

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Shape	Moment of Inertia
Hoop	mr^2
Cylinder	$\frac{1}{2} mr^2$
Rod	$1/12\ mr^2$
Sphere	$2/5\ mr^2$

moment of inertia of a complex object $= \sum I_{parts}$

A: 1
N
O < #30

For a body of many parts its moment of inertia is the sum of the moments of all parts making up the body. Units of moment of inertia are kg m^2

NOTE: that I is dependant on m and r, so two objects of equal mass may have different moments of inertia if they circle about a different radius.

Example: What is the moment of inertia of a 2.0 kg ball of diameter 20 cm swung on the end of a 1.6 m long rod of uniform cross-section and mass 1.2 kg?

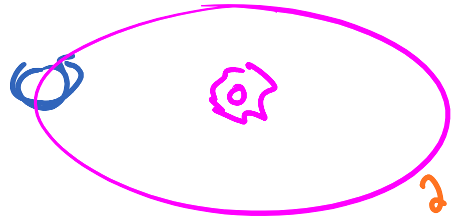
Hoop rad 1.7m
$mr^2 \cdot 5.78$

Angular momentum is the angular analogy of momentum,
symbolized L. It is the product of the moment of inertia [I] (angular mass) and angular velocity [ω].
Units are kg m^2 / s
What is the angular momentum of the Earth about the sun? Include effects of rotation.

$L = I\omega^2$

What is the angular momentum of the Earth about the sun? Include effects of rotation.

$L = I\omega$



$L = \left(mr^2 + \frac{2}{5}mr^2\right)\frac{1}{3.156\times10^7}$

$\left[5.98\times10^{24}(1.5\times10^{11})^2 + \frac{2}{5}(5.98\times10^{24})(6.38\times10^6)^2\right]\frac{2\pi}{3.156\times10^7} = 2.68\times10^{40}\ \frac{kg\,m^2}{s}$

Conservation of Angular Momentum:

Angular momentum is a **conserved quantity**, meaning the total angular momentum of a system must remain constant (assuming no net torque acts) for a body or system.
Consider a rotating mass of moment I and angular velocity ω,
an increase in its mass will increase I, but as L must remain constant ω must decrease.

$L_o = L_f$

$p = mv$

$L = I\omega$

Why does a figure skater rotate faster when they pull their arms inward?

If the radius of orbit of Earth about the sun were magically doubled, what would happen to the angular velocity of the Earth?

Example: a 2.0 kg mass is rotated on a rod of mass 1.0 kg in a horizontal circle of frequency 2.0 Hz, if the rod Is somehow collapsed to 1/2 its length what will be the new frequency?

Posit formula for Rotational K.E.

Autre example:

Earth orbits the sun with period = 1.0 years, what would be the new period if the radius of orbit were halved?

$I_1\omega_1 = L = I_2\omega_2$

$\cancel{m}r^2\omega_1 = \cancel{m}\left(\frac{r}{2}\right)^2\omega_2$

$\omega_1 = \frac{1}{4}\omega_2$

$4\omega_1 = \omega_2$

$T_2 = \frac{T_1}{4}$

$\cancel{m} r_1^2 \omega_1 = \cancel{m}\left(\frac{r_1}{2}\right)^2 \omega_2$

$4\omega_1 = \omega_2$

$\frac{1}{4}$

Example a mass of 250 g is held in a rotating circle with radius 0.50 m on a light (massless) string. If the tangential velocity was 2.0 m/s and the string was to slip to radius 1.0 m what original and new angular velocities would exist?

$m = .25\,kg$

$r = .5\,m$

$I = mr^2$
$= .25(.5)^2$
$= .0625\ kg\,m^2$

$v = \omega r$

$\frac{v}{r} = \omega$

$\frac{2}{.5} = \omega_0 = 4\ \frac{rad}{sec}$

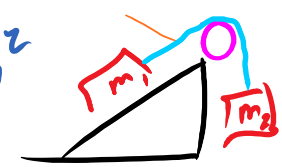
$L = I\omega$
$= .0625(4)$
$= .25\ \frac{kg\,m^2}{s}$

$L = I_f\,\omega_f$
$.25 = m r_f^2\,\omega_f$
$.25 = .25(1)^2\,\omega_f$
$\omega_f = 1.0\ \frac{rad}{sec}$

Rotational Energy
Like all mechanical values there is a rotational kinetic energy just as there is a translational kinetic energy.

Postulate the formula for rotational kinetic energy given that Ek = 1/2 m v^2

$E_{k\,rotational} = \frac{1}{2} I \omega^2$



Complete A:1, A2 TEST Thurs Apr 8th. <= angular stuff, Lens maker stuff, conductive spheres, fundamental forces

~~DO 2 more practice exams over the break.~~

do the review,
Test includes Angular measures, strong/weak nuclear forces, conducting spheres, lens maker eqn., Davisson Germer expt, X-ray production