Projectiles

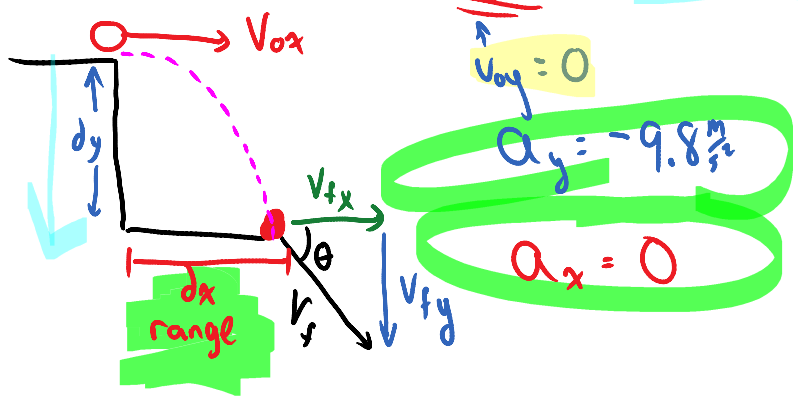
Monday, October 24, 2011
1:56 PM

A projectile is any object projected into the air and freely acted upon by gravitation. There will be 2 components for velocity (x and y directions) and 2 components for displacement (x and y direction).

MAKE UP POSITIVE (down is negative).

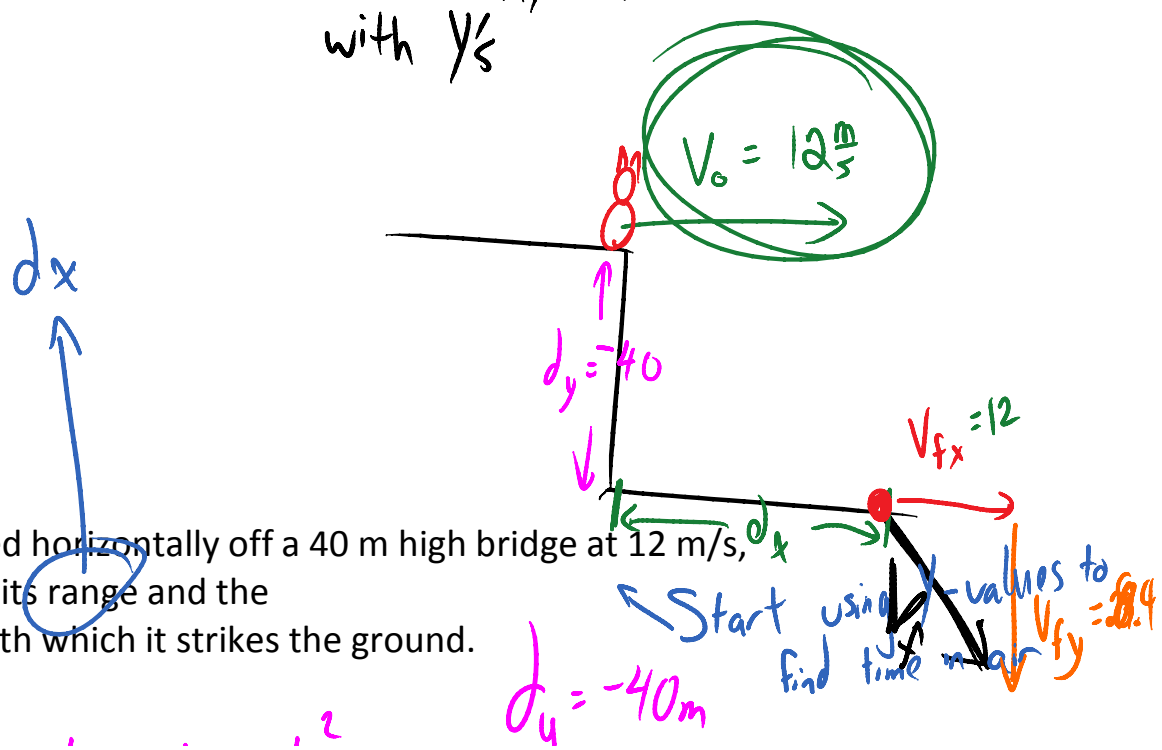
There are 3 types of projectile problems, Type I, Type II and Type III.

Type I has an object projected horizontally off a raised surface



d_y = negative

NEVER MIX X's with Y's



Start using y-values to find time

A cat is fired horizontally off a 40 m high bridge at 12 m/s, determine its range and the Velocity with which it strikes the ground.

velocity with which it strikes the ground.

find time

$d_y = -40m$

$a_y = -9.8 \frac{m}{s^2}$

$V_{oy} = 0$

$t = ?$

$d_y = V_{oy}t + \frac{1}{2}a_y t^2$

$-40 = \cancel{0t} + \frac{1}{2}(-9.8)t^2$

$+40 = +4.9t^2$

$\sqrt{\frac{40}{4.9}} = t = 2.9s$

find range using X-values & time

$d_x = ?$ $V_{ox} = 12\frac{m}{s}$ $a_x = 0$ $t = 2.9s$

$d_x = V_{ox}t + \cancel{\frac{1}{2}a_x t^2}$

$(12)(2.9) = 35m$

$V_{fx} = ?$ $V_{ox} = 12\frac{m}{s}$ $t = 2.9s$ $a_x = 0$

$V_{fx} = V_{ox} + a_x t$

$V_{fx} = 12 + \cancel{0t}$

$V_{fy} = ?$ $V_{oy} = 0$ $a_y = -9.8$ $t = 2.9$

$V_{fy} = \cancel{V_{oy}} + a_y t$

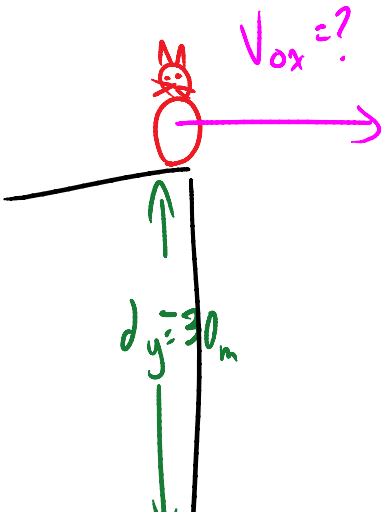
$= -9.8(2.9)$

$= -28.4\frac{m}{s}$

$V_f = 31\frac{m}{s}$

$\theta = \tan^{-1}\left(\frac{28.4}{12}\right) = 67°$ S from E down from horiz.

A cat is fired horizontally and strikes the ground on a spike exactly 15 m from the bottom Of a 30 m high bridge, with what horizontal velocity was it projected?



$d_y = -30$

$V_{oy} = 0$

$a_y = -9.8$

$t = ?$

$V_{ox} = ?$

$d_x = 15$

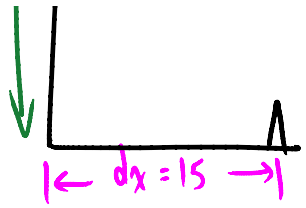
$d = \cancel{V_0 t} + \frac{1}{2}at^2$

$\sqrt{\frac{-30}{4.9}} = \sqrt{+4.9t^2}$

$t = 2.5s$

$d = V_0 t$

$15 = V_0 = 6.0\frac{m}{s}$



V_{ox}

$d_x = 15$

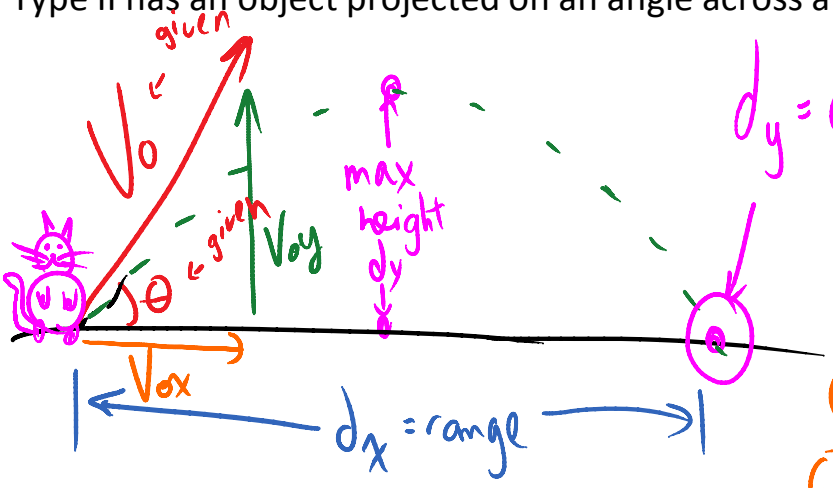
$a_x = 0$

$t = 2.5$

$d = V_0 T$

$\frac{15}{2.5} = V_0 = 6.0 \frac{m}{s}$

Type II has an object projected on an angle across a horizontal surface



$d_y = 0$ ← allows us to use y-comps. to find t

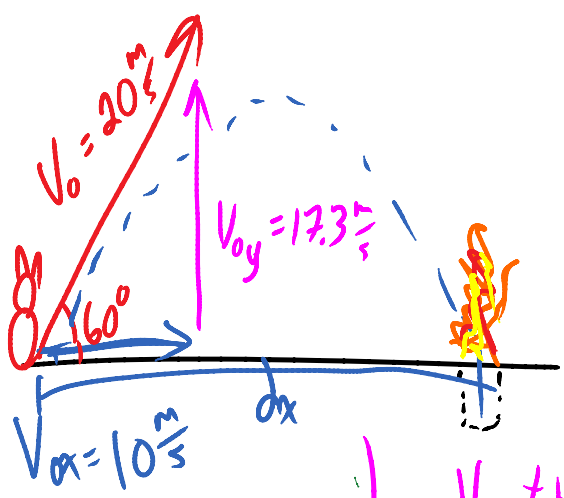
① Break V_0 into components

② $d_y = V_{0y} t + \frac{1}{2} a_y t^2$ to find t

③ find range $d_x = V_{ox} t + \frac{1}{2} a_x t^2$

④ a) find max height

b) find V_f ← V_{fy}, V_{fx} pythag



find range, max height, velocity at impact

$d_y = V_{0y} t + \frac{1}{2} a_y t^2$

$0 = 17.3 t - 4.9 t^2$

$\cancel{4.9} t^{\cancel{2}} = \frac{17.3 \cancel{t}}{4.9}$

$t = 3.5 s$

$d_x = ?$ $a_x = 0$

$V_{ox} = 10$ $t = 3.5$

$d_x = V_{ox} t + \cancel{\frac{1}{2} a_x t^2}$

$= 10(3.5) = 35 m$

Max height ← $V_{fy} = 0$

$d_y = ?$

$V_{oy} = 17.3$

$a_y = -9.8$

$V_{fy}^2 = V_{oy}^2 + 2 a_y d_y$

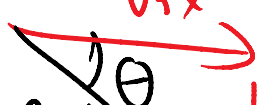
$0^2 = 17.3^2 + 2(-9.8) d_y$

$0 = 300 - 19.6 d_y$

Velocity at impact

$V_{fy} = ?$

$V_{fx} = 10$



$V_{fy} = ?$

$V_{oy} = 17.3$

$a_y = -9.8$

$d_y = 0$

$V_{fy}^2 = 17.3^2 + \cancel{2(-9.8)0}$

$0 = 300 - 19.6 d_y$

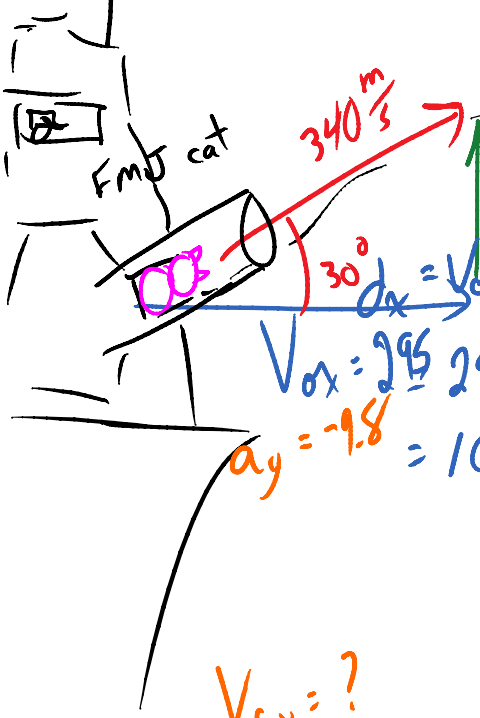
$\cancel{19.6} d_y = \frac{300}{19.6} = 15.3\,m$

$\cancel{19.6} d = \frac{26^2}{19.6}$

$d_y = 34.5\,m$

1, 2, 3, 5, 6 p54

A cat is fired from a battleship at 340 m/s on an angle of 30°, determine the range of the cat and it velocity when it reaches a height of 100m.



$a_y = -9.8$

$V_{fy} = 0$

$d_y = V_{oy}t + \frac{1}{2}a_y t^2$

$0 = 170t - 4.9t^2$

$4.9t^2 = 170t$

$\frac{4.9t^2}{4.9} = \frac{170t}{4.9}$

$t = 34.7\,s$

$d_x = V_{ox}t + \frac{1}{2}a_x t^2$

$V_{ox} = 295$

$295(34.7)$

$= 10215\,m$

$1.02 \times 10^4\,m?$

$10.2k$

$= V_{oy}^2 + 2a_y d_y$

$V_{fy} = ?$

$d_y = 100$

$V_o = 170$

$V_{fy}^2 = 170^2 + 2(-9.8)(100)$

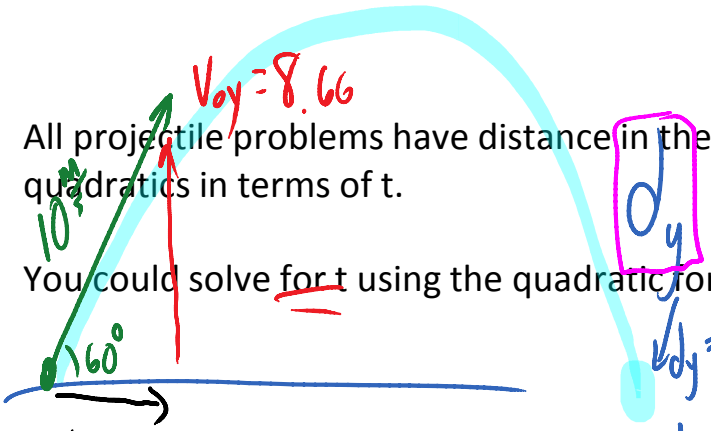
$V_{fy}^2 = 28900 - 1960$

$26940 = \pm 164\,\frac{m}{s}$

1, 2, 3, 5, 6 p54

$V_{fy} = \sqrt{}$

$= V_{oy} t + \frac{1}{2} a_y t^2$



All projectile problems have distance in the Y-direction as a function of t^2. They will be quadratics in terms of t.

You could solve for t using the quadratic formula.

$V_{ox} = 5.00$

$d_x = V_{ox} t + (5)(1.77) = 8.85\ m$

$0 = 8.66t + \frac{1}{2}(-9.8)t^2$

$0 = 8.66t - 4.9t^2$

$4.9t = 8.66$

$t = \frac{8.66}{4.9} = 1.77\ s$

Type II projectiles:

$a_y = -9.8 \frac{m}{s^2}$ ← this slows them as they rise

The key to your success: 1) break all vectors in X and Y components

2) Keep X's separate from Y's

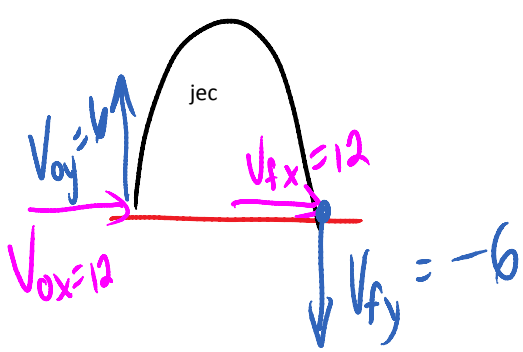
$a_x = 0$ ← this means V_{ox} is constant

Type I example: a cat is thrown horizontally off a 20 m high building. A fire is exactly 4.0 m from base of the building, with what initial velocity must the cat leave the building in order to land in the fire.

Use the Y components to find time in air,
Then use that time to find range, or V_f

V_{ox} exists at the high point! $V_y = 0$ at high point

A cat is kicked at 10.0 m/s at 60° above the horizontal, what is its range?



Finding the velocity of a projectile:

Are you given height d? if you are given d then use the eqn $v_f^2 = v_o^2 + 2\,ad$

Are you given time t? If you are given t then use the eqn $v_f = v_o + at$

Remember Vx never changes!!! Only Vy changes!!!

The same cat as above has what velocity after 1.20 s?

$V_{f_y} = V_{o_y} + a_y t$

$= 8.66 + -9.8(1.2)$

$= -3.1 \frac{m}{s}$

$V_{fx} = 5$

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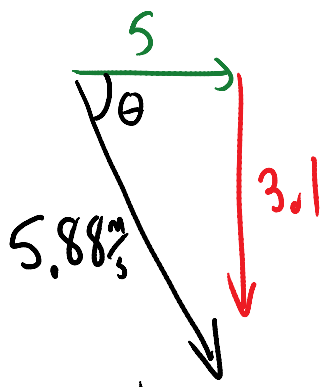
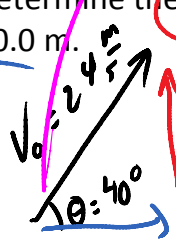


$\theta = \tan^{-1}\left(\frac{3.1}{5}\right) = 32^\circ$ down from Horiz

A A cat is kicked at 24 m/s on an angle of 40° above the horizontal. Determine the range, maximum height and when it reaches height of 10.0 m.



$V_{oy} = 24 \sin 40$
$= 15.4$

$V_{ox} = 24 \cos 40$
$= 18.4$

$d_y \overset{t=?}{=} V_{o_y} t + \frac{1}{2} a_y t^2$

$0 = 15.4t - 4.9t^2$

$\frac{4.9t^2}{4.9t} = \frac{15.4t}{4.9}$

$t = 3.14\ s$

$d_x = V_{ox} t + \cancel{\frac{1}{2} a_x t^2}$

$= (18.4)(3.14)$

$V_{oy} = 15.4 \frac{m}{s}$ $a_y = -9.8 \frac{m}{s^2}$

$= (18.4)(3.14)$
$= 57.8\,m$

$d_y = ?$ $V_{fy} = 0$ $V_{oy} = 15.4\frac{m}{s}$ $a_y = -9.8\frac{m}{s^2}$

$V_{fy}^2 = V_{oy}^2 + 2ad$

$0 = 238 + -19.6d$

$\frac{-19.6d}{-19.6} = \frac{238}{19.6}$

$d = 12.1\,m$

$t = ?$
$d_y = 10\,m$
$V_o = 15.4\,m$
$a_y = -9.8\frac{m}{s^2}$

$d = V_o t + \frac{1}{2}at^2$

$10 = 15.4t - 4.9t^2$

$0 = 15.4t - 4.9t^2 - 10$

$0 = -4.9t^2 + 15.4t - 10$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-15.4 \pm \sqrt{15.4^2 - 4(-4.9)(-10)}}{2(-4.9)}$

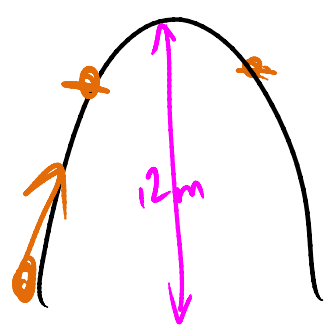
$t = \frac{-15.4 \pm \sqrt{237 - 196}}{-9.8}$ $= V_{fy}$

$t = \frac{-15.4 \pm \sqrt{41}}{-9.8}$

$t = \frac{-V_{oy} \pm V_{fy}}{a}$

$at = -V_o \pm V_f$

$V_o + at = \pm V_f$



type 2

$d = V_o t + \frac{1}{2}at^2$

type 1
=0

find V_{fy} at 10 m height

$V_{fy} = ?$ $a_y = -9.8$

$V_{oy} = 15.4$ $d_y = 10$

$V_f^2 = V_o^2 + 2ad$

$= 15.4^2 - 196$

$V_f^2 = 41$

Type III projectiles are quadratics in terms of t, this means you can find the time in the air using the quadratic formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

OR by finding the Vfy using: $V_{f_y}^2 = V_{o_y}^2 + 2a_y d_y$

Then using $V_f = V_o + at$ ← solve for t